B. Tech. Degree I & II Semester Examination in Marine Engineering May 2013

MRE 101 ENGINEERING MATHEMATICS I

Time : 3 Hours

Maximum Marks : 100

(5 × 20 = 100)

I. (a) Evaluate $\displaystyle\lim_{x \to 0} \frac{e^x - 2\cos x + e^{-x}}{x \sin x}$.

(b) State Rolle's theorem and verify it for the function $f(x) = x - x^3$ in the interval $(-2, 2)$.

(c) If $y = \left\{ \log\left(x + \sqrt{1 + x^2}\right) \right\}^2$, prove that $(1 + x^2) y_{n+2} + (2n + 1) x y_{n+1} + n^2 y_n = 0$.

OR

II. (a) State mean value theorem and use it to find a point c in $(-4, 1)$ for the function $f(x) = x(x + 3) e^{-x/2}$.

(b) If $y = \tan^{-1} x$, prove that $(1 + x^2) y_{x+1} + 2nx y_n + n(n - 1) y_{n-1} = 0$.

III. (a) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r} f'(r)$.

(b) If $u = x^2 \tan^{-1}\left(\dfrac{y}{x}\right) - y^2 \tan^{-1}\left(\dfrac{x}{y}\right)$, find $\dfrac{\partial^2 u}{\partial x \partial y}$.

(c) Examine for extreme values of the function $3x^2 - y^2 + x^3$.

OR

IV. (a) prove that of all the rectangular parellelopipids of given surface, cube has the maximum volume.

(b) Find the percentage error in the area of a rectangle when an error of 1% is made in measuring its length and breadth.

(c) If $u = \dfrac{y^2}{2x}$, $v = \dfrac{x^2 + y^2}{2x}$, find $J\left(\dfrac{u, v}{x, y}\right)$.

(P.T.O.)

V. (a) Derive the standard equation to the parabola in the form $y^2 = 4ax$.

(b) Find the condition that the straight line $\ell x + my + n = 0$ to be normal to the hyperbola
$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

(c) Find the focus and length of latus rectum of the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$.

OR

VI. (a) Find the vertex, focus, directrix and length of the latus rectum of the parabola $y^2 - 8x - 2y + 5 = 0$.

(b) Find the equation of the ellipse whose eccentricity is ½ , one of its foci is $(1, 2)$ and the directrix is $2x - 3y + 6 = 0$.

(c) Find the equation of the hyperbola passing through the point $(3, 4)$ and has the straight line $2x + 3y - 5 = 0$ and $x - 2y + 1 = 0$ as asymptotes.

VII. (a) Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to an extremity of the latus rectum.

(b) Find the area of the Astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(c) Evaluate $\iint xy(x + y)\,dy\,dx$ over the area between $y = x^2$ and $y = x$.

OR

VIII. (a) Change the order and evaluate the integral $\displaystyle\int_{0}^{1}\int_{4y}^{4} e^{x^2}\,dx\,dy$.

(b) Find the volume of a sphere of radius a.

IX. (a) Show that $\overline{a}\times(\overline{b}+\overline{c})+\overline{b}\times(\overline{c}+\overline{a})+\overline{c}\times(\overline{a}+\overline{b})=0$.

(b) Find the unit vector perpendicular to each of the vectors $\overline{A} = 2i - j + k$ and $\overline{B} = 3i + 4j - k$ and obtain the sine of the angle between the vectors $\overline{A}$ and $\overline{B}$.

OR

X. (a) Find a unit vector normal to the surface $z^2 = x^2 + y^2$ at the point $(1, 0, -1)$.

(b) Show that $\overline{F} = (y^2 + 2xz^2)i + (2xy - z)j + (2x^2z - y + 2z)k$ is irrotational and hence find its scalar potential.

(c) Prove that for any vector field V, $div(curl\ V) = 0$.

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